1876MNRAS..36..355B

On a hitherto unnoticed Apparent Inequality in the Longitude of the Moon. By Professor Simon Newcomb.

This inequality was brought to light in the course of an investigation having for its object the determination of the corrections to be applied to the lunar Ephemeris derived from Hansen's *Tables de la Lune*, in order that that Ephemeris might be used for the determination of such of the longitudes of Transit of *Venus* stations as depend on observations of occultations or Moon culminations. The material used was the series of Greenwich and Washington meridian observations of the Moon, extending from 1862 to 1874, and the residual errors of the Moon's place from Hansen's Tables thence derived. These residuals were first corrected for the theoretical terms in which the tables seem to need emendation, and for error of Moon's semi-diameter, and then for the mysterious error of epoch which has been so rapidly developed during the past ten years. The corrections thus applied, except the last, using Hansen's notation, and putting for brevity

$$\mathrm{D} = g - g' + \omega - \omega',$$

which is equal to the mean longitude of the Moon, minus that of the Sun, were as follow:—

For error of semi-diameter		$\pm 1''\cdot 00$	
For correction of parallactic terms		$+0\cdot 96$	sin D
		$-0\cdot 13$	$\sin(\mathrm{D}+g')$
„	annual equation	$+0\cdot 09$	$\sin(g')$
„	accidental error in tables	$-0\cdot 62$	$\sin(2\mathrm{D}-2g'-2\omega')$.

Hansen's increase of his inequalities by the constant factor ·00015, on account of supposed want of coincidence between the centre of gravity and centre of figure of the Moon, was not removed from any of the terms except the annual equation, because reasons are shown why both the evection and the variation should seem larger from meridian observations than from Hansen's theory.

When these several corrections were applied to the residuals, I was surprised to find systematic errors outstanding which could not be represented by corrections to the lunar elements. Of their reality there could be no serious doubt, because the Greenwich and Washington observations agreed in showing them. At first I was disposed to attribute them to inequalities in the surface of the Moon, but a more careful examination showed that they were periodic, and developed on the Moon's longitude, being at first positive when the longitude was between 180° and 360°, and negative in the first semi-circumference. But this was found to be subject to a sensible alteration, the point of

1876MNRAS..36..358N

maximum positive error moving forward to about 0° and that of negative error to 180° in the course of a few years' observations.

It was now evident that, in the investigation of this inequality, its period might, during any one year, be regarded as the same with that of the Moon's mean anomaly, and that thus, during this interval, it would be identified with the correction to the eccentricity and perigee. Owing, however, to the want of exact coincidence of period, the apparent errors of eccentricity and perigee would be different from year to year—these elements would, in fact, appear to be affected with an inequality of long period.

In order to make the investigation more complete I also determined the corresponding corrections for the years 1847 to 1858, for which the residual errors of Hansen's Tables are given in the Greenwich observations for 1859. The resulting corrections and the mode in which the new term is brought out are shown in the following table. The residual errors of Moon's longitude during each term of about a year were supposed to be represented in the form

$$\Delta\, \delta l + h \sin g + k \cos g,$$

g being the mean anomaly, and h and k the coefficients to be determined from observation, on which depends the correction to eccentricity and perigee. Their values in terms of these last are—

$$h = -2\,\delta e$$
$$k = 2e\,\delta\pi.$$

The values of h and k, as they result immediately from observations, are shown in the second and third columns of the following table. Their periodicity is obvious. To make its law more evident, we find by a preliminary treatment that the constant portions of these quantities are approximately—

$$\text{const. of } h = h_0 = +0''{\cdot}50,$$
$$\text{const. of } k = k \;= +0{\cdot}10.$$

Subtracting these portions, we obtain the values of the purely periodic portions, which we call h' and k', and which we present in the next two columns. We now see from these columns that in addition to h' and k' being separately periodic, they have each about the same period, and if one is represented by a cosine, the other will be nearly represented by the sine of the same angle. Let us then determine a and N from the equations

$$a \sin \mathrm{N} = -k',$$
$$a \cos \mathrm{N} = \;\; h'.$$

The values of a and of N thus deduced are shown in the two

1876MNRAS..36..358N

following columns. The regular progression of the angle N is too striking to be overlooked, extending as it does through an interval of 27 years. We wish to see how it may be represented by a quantity increasing uniformly with the time. We first remark that the greater the value of a the more accurately will it be fixed by the observations. On this principle, we assign to N the weight in the column next following. The angle which best represents it, determined by least squares, is given in the column $\mu + nt$. I afterwards concluded that too great weight had been given to the older observations, and finally adopted, as the most probable value of N,

$$N = 163^{\circ}{\cdot}2 + 21^{\circ}{\cdot}6\,(t - 1868{\cdot}5),$$

the residuals from which are given in the last column. The mean values of a are—

$$\text{from the old series, } a = 1''{\cdot}66;$$
$$\text{from the new series, } a = 1{\cdot}55.$$

The effect of the accidental errors of observation is to increase the value of a. I therefore consider that we may put—

$$a = 1''{\cdot}50.$$

[It may be remarked that the values of a and N derived rigorously by least squares from the second series of observations above are—

$$a = 1''{\cdot}52,$$
$$N = 161^{\circ}{\cdot}2 + 22^{\circ}{\cdot}8\,(t - 1868{\cdot}5)].$$

The table referred to is :

Mean Date.	h	k	h'	k'	a	N	Wt.	$\mu+nt$	ΔN	Δ′N
	″	″	″	″	″	°		°	°	°
1847·8	−0·08	+0·55	−0·58	+0·45	0·74	52	1	94	+ 42	+ 24
48·9	−0·55	−1·38	−1·05	−1·48	1·82	145	3	118	− 2	− 44
50·1	−0·20	−1·91	−0·70	−2·01	2·13	161	3	141	− 20	− 36
51·2	−0·32	−1·92	−0·82	−2·02	2·18	158	3	165	+ 7	− 8
52·4	+0·26	−2·45	−0·24	−2·55	2·56	175	4	189	+ 14	0
53·5	+1·10	−1·88	+0·60	−1·98	2·07	197	3	212	+ 15	+ 2
54·6	+1·45	−1·40	+0·95	−1·50	1·77	212	3	236	+ 24	+ 12
55·8	+0·77	+0·31	+0·27	+0·21	0·34	308	½	260	− 48	− 60
56·9	+1·76	+1·82	+1·26	+1·72	2·13	328	3	284	− 44	− 55
1858·1	−0·17	+0·66	−0·67	+0·56	0·88	50	1	307	−103	−112

1876MNRAS..36..358N

Mean Date	*h*	*k*	*h′*	*k′*	*a*	N	Wt.	$\mu+nt$	ΔN	Δ′N
	″	″	″	″	″	°		°	°	°
1862·5	+0·04	+1·23	−0·46	+1·13	1·22	22	3	40	+18	+12
63·5	−0·64	+1·78	−1·14	+1·68	2·03	34	5	61	+27	+21
64·5	−1·07	+1·09	−1·57	+0·99	1·85	58	5	81	+23	+19
65·5	−1·03	−0·15	−1·53	−0·25	1·55	99	4	102	+3	−1
66·5	−0·47	+0·10	−0·97	0·00	0·97	90	2	123	+33	+30
67·5	−1·93	−0·36	−1·43	−0·46	1·50	108	4	144	+36	+34
68·5	+0·34	−1·46	−0·16	−1·56	1·57	174	4	165	−9	−11
69·5	+1·67	−1·56	+1·17	−1·66	2·03	215	5	185	−30	−30
70·5	+1·48	−1·14	+0·98	−1·24	1·58	218	5	206	−12	−12
71·5	+1·65	−0·36	+1·15	−0·46	1·24	248	3	227	−21	−20
72·5	+2·15	−0·12	+1·65	−0·22	1·66	262	4	248	−14	−12
73·5	+1·91	+0·16	+1·41	+0·06	1·41	272	4	269	−3	−1
1874·5	+1·92	+0·60	+1·42	+0·50	1·50	289	4	289	0	+4

The corrections to the Moon's longitude arising from this term is—

$$\begin{aligned}\delta v &= -h' \sin g - k' \cos v \\ &= -1''{\cdot}50\,(\sin \mathrm{N} \sin g - \cos \mathrm{N} \cos g) \\ &= -1''{\cdot}50 \sin (g + \mathrm{N} - 90^\circ),\end{aligned}$$

which is the hitherto unnoticed apparent inequality of the Moon's longitude.

The period of N is $16\frac{2}{3}$ years, with a probable error of about half a year, and a possible error of a year or more. The corresponding period of N + *g*, or of the inequality in longitude, is—

$$27{\cdot}4304 \text{ days} \pm 0^{\mathrm{d}}{\cdot}0040.$$

There is a large preponderance of probabilities against the real period being either less than $27^{\mathrm{d}}.42$ or more than $27^{\mathrm{d}}.44$. No known term in the Moon's longitude falls between these limits. The Moon's sidereal period is $27^{\mathrm{d}}.32$, and her anomalistic period is $27^{\mathrm{d}}.55$, so that the new term falls half-way between these two.

The non-accordance of this period with that of any term heretofore sought for is the reason why this term has not before been noticed. A term of unknown period would not be remarked unless its magnitude was such as to visibly affect the individual comparisons of theory with observations; and Hansen's Tables, as corrected, are the first ones of which the residual errors are so small that a term of $1''{\cdot}5$ would be remarked in the comparison with observations.